A Dynamic Model for an Optimal Price Promotion Strategy of New Products for Retailers

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ABSTRACT

In this paper, we intend to create a dynamic model through a simple mathematical assumption by applying the variables of time and promotion expenses to linear demand equation in economics. In the process of research, we use the Euler-Lagrange equation in the Calculus of Variations in an attempt to develop a dynamic model for an optimal price promotionstrategy. Finally, after having gotten the best equation, we adopt the sensitivity analysis to find out the correlation between decision variables and parameters. The findings of the current study can serve as an aid in improving the basis for management decision making.

Keywords: Euler-Lagrange equations, dynamic model, retailer, price promotion, sensitivity analysis

INTRODUCTION

Sales promotion in marketing management includes a wide range of selling cause tools, most of which are of short-term nature. The aim is to stimulate an immediate or desired purchase response to a specific product or service from targeted customers or distributors. Therefore, different promotion objectives and goals require different advertising appeals. The main objective of advertising is to offer reasons to get target audience to make purchases, whereas the sales promotion provides extra incentives for buying the product. Sellers use a diverse collection of incentives to attract new and potential clients, to reward for loyal customers, as well as to increase the rate of return buying of occasional users.

A decade ago, the advertising to sales promotion ratio was about 60:40. Nowadays the situation has changed dramatically. Today in many companies which sell consumer products, the sales promotion budget accounts for 65 percent to 75 percent of the combined A & P expenditures (Kotler, 2006). In recent decades, the budget expenditures for sales promotion have increased significantly every year. The rate of fast growth is expected to continue. As to advertising spending, the rapid growth in multi-channel operators such as cable has made a big impact on advertising, which only indicates the importance of promotion activities. Counghlan et al. (2001) viewed that sales promotion can yield profits for manufacturers and consumers alike.

Sales promotions are primary short-term incentives that are offered to consumers and trade channel members to stimulate consumer demand and improve dealer effectiveness. Sales promotions involve sales promotion to consumers, sales promotion to traders, and promotion for sales representatives. Consumer promotions include samples, coupons, discounts, cash refund offers, presents, patronage rewards, free trials, promotion linked to additional products and point-of-purchase displays. Trade promotions include discounts, gratis goods, advertising and display budget, cooperative advertising, trade incentives and trade contests. Promotions for sales representatives include bonuses for outstanding salesmen, exhibitions, conferences and competitions among sales staff (Kotler, 2006). These incentives are generally used as means by many companies, and in addition, manufacturers, distributors and retailers to boost sales.

Among the most common promotion practices are 「price reduction」 and 「discount coupons」. According to the research findings by Alvarez & Casielles (2005), the most effective sales promotion is immediate price reduction. The consumer is the one who can squander money on a more expensive brand which he would like to buy. Nevertheless, he can choose to buy an alternative product at a lower cost. Therefore, the price reduction strategy is the best sales promotion move. In a research study by Chen, Monroe, and Lou (1998), they claim that compared to the price reduction tactic, consumers hold a widely differing view on discount coupons. Consumers perceive coupons as a temporary special offer rather than a price reduction. Coupons that are issued for established brands tend to be redeemed primarily by individuals who would normally purchase the product anyway. Thus, regular customers will be the ones who enjoy the benefit of savings. To new or trial users, they would have to pay the full price for the initial purchase of a product or service. However, since the product is bought at the regular price, consumers are less likely to feel that the quality of the product has reduced.

A marketing channel consists of different channel members, each having its own decision variables. However, each channel member's decisions do affect the other channel members' profits and actions (Jeuland and Shuagn, 1983). A lack of coordination in the channel can lead to undesirable results. In a fierce competitive environment, keeping an eye on the competitors' move is very important. A move late in time can result in disastrous consequences. For instance, a retailer may deploy a price discount tactic with the aim to increase the sales volume for the business. Fearing that their channel partner will draw customers away, the other competing members of the distribution channel will have to do the same by launching an even more competitive promotional tool. As far as the manufacturers are concerned, if the manufacturer fails to comply with the channel members' demand for more trade support, the retailer may not want to carry the product or make an extra effort to push it to customers, the consequence of which can cause a big sales drop in business. This example illustrates the point that the retailers have significant and growing power over the producers or manufacturers.

"Those who maintain control over channel marketing are the winners," said Bill Gates, the founder of the Microsoft. This famous saying highlights the importance of distribution channel, which is the key secret to business success. To reach end users, businesses need a well-knitted network. A well networked and properly knitted distribution channel plays an important role in the enhancement of sales growth. Let's take the downstream retailers such as convenience stores, supermarkets and mass markets in the retail industry in Taiwan for example. With consolidation, retailers have gained access to sophisticated information. For example, the use of computers and bar codes on packages is shifting the balance of power in their favor. Today, not only do the retailers have more bargaining power, but they have the capacity to determine the volume of sales for their products. Hence, efforts made by the retailers for generating sales have an important influence on the demands of consumers. Manufacturers can otherwise make good use of product displays, billboard advertising and shows and demonstrations to increase sales (Sudhir, 2001).

Most research in the area of marketing channels has focused primarily on developing models that determine optimal single period decisions for channel members (Dolan, 1987), such as the relationship between the manufacturers and retailers. These static models do no account for the effect of current period marketing decisions of channel members on their future actions. While understanding the dynamic behavior of channel members is important for designing effective marketing strategies, this issue has received limited attention in the literature. However, the studies by Sudhir (2001) and Kotler (2006) and Gensler et al. (2007) proposed the dynamic pricing behavior of channel members. Besides determining dynamic pricing strategies, it is equally important for channel members to determine their optimal levels

of marketing effort (e.g. advertising expenditures, promotion fee) over time, as effort in any period affects sales not only in that period, but also in future periods as well (Dhrymes 1971, Blattberg and Jeuland 1981). Business executives and decision makers must be in total control of the timing of the marketing campaign so as to help them make the optimal decisions.

Since much of the previous research in marketing strategy models has tended to focus on the static models of strategy and performance, there is a need for dynamic models to capture such lagged effects of marketing effort. According to some research findings, price promotion works most effectively among consumers. This is because it is very effective in drawing attention and giving the consumer the feeling of having saved money. The purpose of this research is to construct a dynamic model for an optimal price promotion strategy by means of a simple mathematical model through the application of time and promotion expenses variables to linear demand equation. Also, we attempt to investigate the effects of marketing parameters on the decision-making at the managerial level. The result of this study can serve as an aid in improving the basis for management decision making.

SYMBOLS AND ASSUMPTIONS

Variables in this study are defined as follows:

a, b: positive constants

c : manufacturing cost per unit for a new product

$p(t)$: unit price of a new product at time t

$q(t)$: demand of new products at time t

r : discount rate

t : time, $t \in [0, T]$ any time during new products promotion period

$[0, T]$: new products promotion period

w : manufacturer's wholesale price for retailers

$x(t)$: cumulated units sold in the period of $[0, t]$

$x'(t)$: units sold of new products at time t

$x(T)$: cumulated units sold in the period of $[0, T]$

π_r: retailer profits of new products

π_m: manufacturer profits of new products

v : sales promotion expenses per unit sold offered by manufacturers to retailers

δ: the ratio of sales promotion cost at stores to sales promotion expenses per unit $(0 \leq \delta \leq 1)$

λ : the ratio of the discount amount to sales promotion expenses per unit $(0 \leq \lambda \leq 1)$

For retailers, the symbols listed above—a, b, c, w, T, v are known as parameters, $x(t), x(T), p(t)$ as decision variables, and δ, λ as positive constants.

BASIC ASSUMYTIONS

This study intends to develop a bilateral monopoly model for channel competition in which an upstream manufacturer sells a new product to a single independent downstream distributor or retailer. This bilateral monopoly framework can be illustrated as below. Other basic assumptions are made as follows:
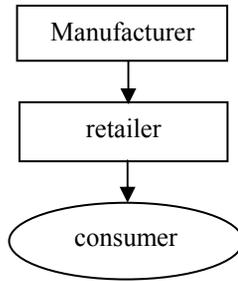
Figure1: A Competitive Framework for Monopoly Manufacture & Retailer

1. A company seeks to maximize its total profit.

2. In competitive markets, the demand curve is often graphed as a straight line with negative slope, might have the form

$$q = a - bp \qquad (1)$$

where a, b are positive parameters a denoting the size of potential markets and b the degree of substitution for a product. Considering the time factor, namely, dynamic, we examine retailer $[0, T]$ during new products promotion period. (1) is modified into symbols $q(t)$ and $p(t)$, representing respectively the units sold and the price of a new product. The units sold by retailers at time t is

$$q(t) = a - bp(t) \qquad (2)$$

For the sake of convenience, hereinafter, we replace $q(t)$ with $x(t)$ which denotescumulated units soldduring $[0, t]$, $x(0) = 0$, $x'(t) = \frac{dx(t)}{dt}$ representing units sold of a new product at time t, replacing $q(t)$ by $x'(t)$ in (2), we obtain

$$x'(t) = a - bp(t) \qquad (3)$$

or, equivalently

$$p(t) = \frac{x'(t)}{-b} + \frac{a}{b} \qquad (4)$$

If there is no promotion campaign sponsored by the manufacturer, new products will be sold at the wholesale price w to the retailer. Assuming the retailer has the control over the price and has zero sales cost, the profit of the retailer is π_r

$$\pi_r = (p(t) - w)x'(t) = \left(\frac{x'(t)}{-b} + \frac{a}{b} - w\right)x'(t) \qquad (5)$$

The manufacturer's profitsπ_m can be presented as

$$\pi_m = (w - c)x'(t) \qquad (6)$$

MATHEMATICAL MODEL

We assume that both manufacturers and retailers make independent pricing decisions to maximize their own respective profits. Due to the unfair asymmetric information available to upstream and downstream suppliers, we model the decision-making measure of a supply channel with one manufacturer and one retailer as a Stackelberg game, where manufacturers are leaders while retailers are followers. As leaders are familiar with the reaction function of the followers, they are able to substitute leaders' profit function with followers' price reaction function. Leaders set their wholesale price based on maximum profits, and then followers' pricing strategies are made accordingly by reacting to the leaders' wholesale price.
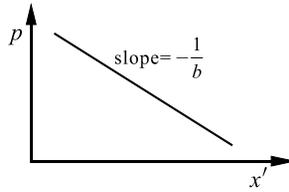
If the retailer takes a proportion of δ from the sales promotion expenses and uses it as the cost at stores for promoting a new product, implying that $\delta \cdot v \cdot x'(t)$ is the amount of money for proceeding new product promotion activities (e.g. displaying costs at stores, POP, slotting fees, etc.).The retailer, after deducting sales promotion costs at stores, obtains $(1-\delta) \cdot v \cdot x'(t)$ as the remaining balance retailers obtain after sales promotion expenses.Simultaneously, in line with the sales promotion activities, the retailer offers consumers with a discount as a feedback. The discount is calculated based on the amount of sales promotion expenses per unit provided by the manufacturer. If the discount scope is λ, λv is the discount amount per unit of a new product. A graph is given below to illustrate the allocation of the sales promotion expenses v of a new product:

$v \cdot x'(t)$:sales promotion expenses at time t

δ

the ratio of sales promotion cost at stores to sales promotion expenses per unit

λ

the ratio of the discount amount to sales promotion expenses per unit

$\downarrow$

$\delta \cdot v \cdot x'(t)$	$(1-\delta) \cdot v \cdot x'(t)$	λv
the promotion expenses at stores	the remaining balance retailers obtain after sales promotion expenses	the discount amount per unit of a new product

Figure 2: the ratio of total sales promotion expenses

In figure 2 above, the discount amount per unit price is given by $\frac{\lambda v x'(t)}{x'(t)} = \lambda v$, where $\lambda v x'(t)$ represents the total amount of the reduced retail price of a new product at time t, while v denotes the sales promotion expenses per unit sold offered by manufacturers to retailers.

Given that a manufacturer requests a retailer to display its new product at stores in order to attract consumers' attention, to do so, as an encouragement, the manufacturer provides the retailer with a v , which denotes sales promotion expenses per unit sold by the retailer for a new product during the promotion period. The term $v \cdot x(t)$ represents the total amount sold of a new product in the period of time t.

When v (the sales promotion expenses per unit sold offered by manufacturers to retailers) and w (the wholesale price) are set by the manufacturer, and a discount price is provided to consumers by the retailer, the term $p(t) - \lambda v$ is used to express the retail price of a new product set by the retailer. Hence, with the discount price, we rearrange the demand function(3) to (7) as shown below:

$$x'(t) = a - b(p(t) - \lambda v) \tag{7}$$

And the new retail unit price $p(t)$ after discount for a new product is given by

$$p(t) = \left(\frac{x'(t)}{-b} + \frac{a}{b} + \lambda v \right) \tag{8}$$

Observing (8), $\frac{\partial P}{\partial x'} = -\frac{1}{b} < 0$ implies lower prices are associated with the increase of sales volumes.The relation is shown in figure 3.

Figure 3: Demand Curve

By (7) and (8), we obtain π_r, which denotes the profit of the new retail price for the new product after discount.

$$\pi_r = \left(\frac{x'(t)}{-b} + \frac{a}{b} + \lambda v - w\right)x'(t) + v(1-\delta)x'(t)$$

$$= \left(\frac{x'(t)}{-b} + \frac{a}{b} - w + (1-\delta+\lambda)v\right)x'(t) \tag{9}$$

By observing (9), π_r is a quadratic function with $x'(t)$ as its variable. The graphic for it is aparabola, and theterm $\frac{\partial^2 \pi_r}{\partial(x')^2} = -\frac{1}{b} < 0$ implies the curve is concave downward. Hence, as shown in figure 4, there exists a critical point $x_c' \in [0,T]$, such that x_c' attains an maximum of π_r, indicating the existence of an appropriate sales amount allowing maximum profits for retailers' new product after discount being exercised.
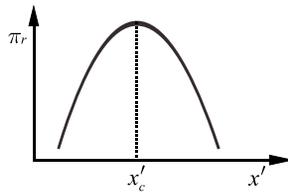
Figure 4: A Downward Parabola Manifesting a Maximum Value

The profits of the manufacturer π_m might have the form

$$\pi_m = (w - c - v)x'(t) \tag{10}$$

During the sales promotion period $[0,T]$ of a new product, we use aforementioned symbols and assumptions and consider the discount rate as well to solve the problem that retailers face. By applying (9), the problem that retailers face under the set v and w is to seek an optimal $x^*(t)$ in order to maximize retailers' sales promotion profit. The mathematical model is hence given by

$$\pi_r^* = \max_x \int_0^T e^{-rt}\left[\frac{x'(t)}{-b} + \frac{a}{b} - w + (1-\delta+\lambda)v\right]x'(t)dt \tag{11}$$

$$\text{Subject to} x(0) = 0, x'(t) \geq 0$$

With seekingthe maximum sales promotion profits of a new product as the primary consideration in mind , manufacturers take into account retailers' reactions (as presented in model 11) when choosing the optimal promotion expenses for a new product v. The mathematical model is given below:

$$\pi_m^* = \max_v \int_0^T e^{-rt}(w - c - v)x'(t)dt \tag{12}$$

DYNAMIC OPTIMALSOLUTION

Theorem A The optimal $x'(t)$ of a new product for retailers at time t is

$$x^{*'}(t) = \frac{b}{2}\left[\frac{a}{b} - w + (1 - \delta + \lambda)v\right]$$

Proof Consider the integrand of functional (11), let

$$F(t, x, x') = e^{-rt}\left[\frac{x'(t)}{-b} + \frac{a}{b} - w + (1 - \delta + \lambda)v\right]x'(t)$$

A necessary condition of (11) to attain an extreme value is that the extremizing function $x(t)$ should satisfy Euler-Lagrange equation

$$\frac{\partial F}{\partial x} = \frac{d}{dt}\left(\frac{\partial F}{\partial x'}\right) \tag{13}$$

Since F is independent of x, then $\frac{\partial F}{\partial x} = 0$

$$\frac{\partial F}{\partial x'} = e^{-rt}\left[\frac{2x'(t)}{-b} + \frac{a}{b} - w + (1 - \delta + \lambda)v\right] \tag{14}$$

Substituting (14) into Euler-Lagrange equation(13), we derive

$$0 = -re^{-rt}\left[\frac{2x'(t)}{-b} + \frac{a}{b} - w + (1 - \delta + \lambda)v\right] + e^{-rt}\left(\frac{2x''(t)}{-b}\right)$$

Which then is reduced to

$$0 = \frac{-2re^{-rt}x'(t) + 2e^{-rt}x''(t)}{-b} + \frac{-rae^{-rt}}{b} + rwe^{-rt} - r(1 - \delta + \lambda)ve^{-rt}$$

$$0 = \frac{-2rx'(t) + 2x''(t)}{-b} + \frac{-ra}{b} + rw - r(1 - \delta + \lambda)ve^{-rt}(\because e^{-rt} \neq 0)$$

$$\frac{-2rx'(t) + 2x''(t)}{-b} = r\left[\frac{a}{b} - w + (1 - \delta + \lambda)v\right]$$

Therefore, the equation that we want to maximize is a second order differential equation

$$x''(t) - rx'(t) = \frac{-rb}{2}\left[\frac{a}{b} - w + (1 - \delta + \lambda)v\right] \tag{15}$$

Let $x'(t) = u$, then $x''(t) = u'$.Replacing $x'(t)$ and $x''(t)$ in (15) u and u', we obtain a general first-order linear equation. Then we multiply both sides by the integration factor $e^{\int(-r)dt} = e^{-rt}$

$$e^{-rt}u' - re^{-rt}u = \frac{-rb}{2}\left[\frac{a}{b} - w + (1 - \delta + \lambda)v\right]e^{-rt}$$

The equation takes the form

$$\frac{d}{dt}(e^{-rt}u) = \frac{-rb}{2}\left[\frac{a}{b} - w + (1 - \delta + \lambda)v\right]e^{-rt}$$

Integration of both sides yields

$$e^{-rt}u = \frac{-rb}{2}\left[\frac{a}{b} - w + (1 - \delta + \lambda)v\right]\frac{1}{-r}e^{-rt} + k \qquad \text{where } k \text{ is a constant}$$

and thus

$$x'(t) = u = \frac{b}{2}\left[\frac{a}{b} - w + (1 - \delta + \lambda)v\right] + ke^{rt}$$

The optimal solution is

$$x^{*'}(t) = \frac{b}{2}\left[\frac{a}{b} - w + (1 - \delta + \lambda)v\right] + ke^{rt} \tag{16}$$

Integrating with respect to t, then

$$x^{*}(t) = \frac{b}{2}\left[\frac{a}{b} - w + (1 - \delta + \lambda)v\right]t + \frac{k}{r}e^{rt} + l \tag{17}$$

where l is an another constant, by boundary condition

$$x(0) = 0 \quad \Rightarrow \frac{k}{r} + l = 0 \quad \Rightarrow \quad l = -\frac{k}{r} \tag{18}$$

Substituting(17) in (18), we obtain

$$x^*(t) = \frac{b}{2}\left[\frac{a}{b} - w + (1 - \delta + \lambda)v\right]t + \frac{k_1}{r}(e^{rt} - 1)$$

by letting $t = T$, then

$$x^*(T) = \frac{b}{2}\left[\frac{a}{b} - w + (1 - \delta + \lambda)v\right]T + \frac{k_1}{r}(e^{rT} - 1) \tag{19}$$

In the optimization problem, a transversality condition describes what must be satisfied at the end of the time horizon. i.e. If the terminal condition is that x' can take on any value, then it must be that the marginal value of a change in x' must equal to zero, i.e.,

$$\left.\frac{\partial F}{\partial x'}\right|_T = 0$$

$$e^{-rT}\left[\frac{2x'(T)}{-b} + \frac{a}{b} - w + (1 - \delta + \lambda)v\right] = 0$$

or, equivalently,

$$x'(T) = \frac{b}{2}\left[\frac{a}{b} - w + (1 - \delta + \lambda)v\right] \tag{20}$$

Putting $t = T$ in (16) and comparing (20)，we know that $k = 0$.By (16)、(17), we conclude the optimal $x'(t)$ is

$$x^{*\prime}(t) = \frac{b}{2}\left[\frac{a}{b} - w + (1 - \delta + \lambda)v\right] \tag{21}$$

Integrating $x'(t)$ with respect to t, the cumulatedunits we get in the promotion period are

$$x^*(T) = \frac{b}{2}\left[\frac{a}{b} - w + (1 - \delta + \lambda)v\right]T \tag{22}$$

Theorem B The optimal price of a new product per unit at time t is

$$p^*(t) = \frac{b}{2}\left[\frac{a}{b} + w - (1 - \delta - \lambda)v\right]$$

*Proof*Substituting (21) into (8),

$$p(t) = \frac{1}{-2}\left[\frac{a}{b} - w + (1 - \delta + \lambda)v\right] + \frac{a}{b} + \lambda v$$

We get

$$p^*(t) = \frac{b}{2}\left[\frac{a}{b} + w - (1 - \delta - \lambda)v\right] \tag{23}$$

Theorem C Manufacturers' optimal sales promotion expense of a new product is given by

$$v^* = \frac{bw + bwy - a - bcy}{2by}$$

$$\text{where } y = 1 - \delta + \lambda \tag{24}$$

Proof Substituting (21) into (12)，the profit object function is

$$\pi_m = \int_0^T e^{-rt}(w - c - v)\frac{b}{2}\left[\frac{a}{b} - w + (1 - \delta + \lambda)v\right]dt$$

Consider the integrand of π_m, let

$$G(t, v) = e^{-rt}(w - c - v)\frac{b}{2}\left[\frac{a}{b} - w + (1 - \delta + \lambda)v\right]$$

Replacing $(1 - \delta + \lambda)$ by y gives

$$G(t,v) = e^{-rt}(w - c - v)\frac{b}{2}\left[\frac{a}{b} - w + yv\right]$$

Since π_m attains an extremum, then

$$\frac{\partial \pi_m}{\partial v} = 0$$

Which implies that

$$\frac{\partial}{\partial v}\int_0^T G(t,v)dt = \int_0^T \frac{\partial}{\partial v}G(t,v)dt = 0$$

Using the product rule of derivatives, we differentiate $G(t,v)$ partially with respect to v, and then obtain

$$\frac{\partial G}{\partial v} = e^{-rt}(-1)\frac{b}{2}\left[\frac{a}{b} - w + yv\right] + e^{-rt}(w - c - v)\frac{b}{2}y$$

$$= e^{-rt}\frac{b}{2}\left(-\frac{a}{b} + w - yv + yw - yc - yv\right) \quad (25)$$

Integrating between 0 to T produces

$$\int_0^T e^{-rt}\frac{b}{2}\left(-\frac{a}{b} + w - yv + yw - yc - yv\right)dt = 0$$

$$\frac{1}{-2r}(-a + bw + byw - byc - 2byv)e^{-rT} + \frac{1}{2r}(-a + bw + byw - byc - 2byv) = 0$$

We conclude

$$-a + bw + bwy - byc - 2bvy = 0$$

Solving this equation for v, we get

$$v^* = \frac{bw + bwy - a - bcy}{2by} \quad (26)$$

Theorem D Retailers' optimal sales promotion profit of a new product is

$$\pi_r^* = \frac{b}{4}\left(\frac{a}{b} - w + yv\right)^2 \frac{1}{-r}(e^{-rT} - 1)$$

$$\text{where } y = 1 - \delta + \lambda$$

Proof According to (11), the total profit of the retailer is

$$\pi_r = \int_0^T e^{-rt}\left[\frac{x'(t)}{-b} + \frac{a}{b} - w + (1 - \delta + \lambda)v\right]x'(t)dt$$

Replacing $(1 - \delta + \lambda)$ by y gives

$$\pi_r = \int_0^T e^{-rt}\left\{\frac{[x'(t)]^2}{-b} + \left(\frac{a}{b} - w + yv\right)x'(t)\right\}dt \quad (27)$$

By applying theorem A to (27), we see that

$$x^{*'}(t) = \frac{b}{2}\left[\frac{a}{b} - w + (1 - \delta + \lambda)v\right] = \frac{b}{2}\left[\frac{a}{b} - w + yv\right]$$

Substituting $x^{*'}(t)$ into (27), we get

$$\pi_r^* = \int_0^T e^{-rt}\left[\frac{b}{-4}\left(\frac{a}{b} - w + yv\right)^2 + \frac{b}{2}\left(\frac{a}{b} - w + yv\right)^2\right]dt = \int_0^T e^{-rt}\frac{b}{4}\left(\frac{a}{b} - w + yv\right)^2 dt$$

$$= \frac{b}{4}\left(\frac{a}{b} - w + yv\right)^2 \frac{1}{-r}(e^{-rT} - 1) \quad (28)$$

Based on the derived optimal solution, this study intends to examine the effects of the change of different parameters on the decision variables through sensitivity analysis. The possible aspects oftheeffects are presented in Table 1:

Table 1: Sensitivity Analysis of Parameters and Decision Variables

Decision variables	parameters					Reference Equation
	a	c	w	r	T	
x^*	↑	−	↓	−	↑	(22)
p^*	↑	−	↑	−	−	(23)
π_r^*	↑	↓	↓	↓	↑	(28)

"↑": The effect of parameters on the direction of decision variables is positive.
"↓": The effect of parameters on the direction of decision variables is negative.
"−":Parameters do not have any impact on the direction of decision variables.

CONCLUSIONS

As much of the previous research in sales promotion models of new products is mostly focused on the static models of strategy and performance, this paper adopts an optimal controlled mathematical model which employs Euler-Lagrange equations to acquirea dynamic model for an optimal price promotion strategy. Through equation(28), we figure out the impact of market potential a on the profit of retailers' sales promotion activities for a new product. When an enlargement of a contributes to increasing the profit due to the sales promotion activities of new products, manufacturers' production costs are simultaneously bound to pass on to the wholesale price w. The wholesale price accordingly will have an impact on decreasing the profit of retailers' sales promotion activities for a new product. In addition, the higher the discount rate is, the lower the profit of retailers' new products promotion activities will be. Furthermore, the longer the interval period T of promotion activities is, the more beneficial for retailers' profit gains for new product sales promotion activities will be.

Based on the above derivations and discussions, the findings of this studycontribute to propose hereinafter a number of marketing management implications for decision-makers at managerial level.

1. Retailers can take advantage of their growing strength in holding a strong channel (such as 7-11, CVS, etc.), and request manufacturers to offer other types of allowances for new product promotion activities, such as gifts for customers or slotting fees.

2. As market potential volumes and discount rates are exogenous variables and cannot be manipulated by either manufacturers or retailers, they should be taken into account as factors while deciding on the timing for new product sales promotion activities.

3. The increase of retailers' profitability is associated with the length of time for new product sales promotion activities. Itimplies that the longer the length of time for new product sales promotion activities is, the more beneficial it will be for retailers.

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